Utility Based Regional Purchasing Power Parities

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Abstract

Calculation of purchasing power parities based on observed consumer behaviour typically relies on the existence of some kind of 'international preferences'. We argue that this unrealistic assumption can be abandoned in 'regional' studies of purchasing power, where prices in both territories under study might differ only slightly. Drawing on recent results for the derivation of equivalence scales, we develop a model of regional purchasing power and discuss several procedures for testing model specification. The model is applied to data of the German Income and Expenditure Survey 1998 to estimate the wedge between purchasing power in the western and eastern part of Germany.

JEL: C43, D12, F01

1 Introduction

Purchasing power parities (ppp) are mainly used to either render international comparisons of monetary aggregates (like the GDP) possible or to measure differences in the cost of living across countries. In this paper we focus on the latter use of ppp. For the calculation of ppp, (price) indices of the Laspeyres or Paasche type are widely used. Purchasing power is measured by comparing the value of a particular commodity basket in the base country and abroad. From the 'cost of living' perspective this approach is frequently critisized for two reasons: On the one hand, the choice of any commodity basket (assumed to be consumed in both of the countries) is arbitrary to some extent. On the other hand, focussing on a single commodity basket ignores substitution effects in consumption and leads to biased results. The latter problem may be overcome by 'averaging' the Laspeyres and the Paasche scheme[1] of weighing prices by quantities, preferably by using the Törnqvist index as an approximation to the 'true cost of living index'; see Deaton and Muellbauer (1980), pp. 169–175, and the references given there. Even with these corrections, calculation of ppp implicitly rests on the assumption, that the domestic household's demand, but not his preference ordering (or utility function) changes when he is confronted with the 'new', i.e. foreign, price regime.[2] This is unlikely to be the case. It is far more realistic to assume that the preference ordering of the household changes with the environmental endowment he faces, say climate, geographical situation, provision of non market and public goods etc.[3] Hence, it makes a

[1]Concerning ppp, an index of the Laspeyres type uses a 'typical domestic' quantity basket, and employing a 'typical foreign' commodity basket can be interpreted as application of a Paasche index.

[2]The following point of critique also applies to models assuming some kind of 'international preferences' identical across countries. For example, Neary and Gleeson (1997) assume that international preferences can be represented by a unique Stone-Geary utility functions, hence household demands follow a Linear Expenditure System. See Balk (2001) pp. 32–36 for a general discussion of this strand of research.

[3]One may distinguish between unconditional preferences and preferences conditional on the household's environmental endowment. Only the former may be invariant to the household's location. In what follows, we assume that the 'basic' utility a household derives from a particular environmental endowment per se is the same at any location, whereas changes

difference for the domestic consumer whether she faces foreign prices at home or abroad. Her consumption pattern will be different in both situations, and only incorporation of the latter situation leads to a meaningful calculation of ppp.

The problem can be clarified using more formal notation. Let $C(z, p, U)$ denote the household's cost function. p is the vector of prices for market goods, and z are 'taste shifters' comprising socio-demographic characteristics like number of persons living in the household, age of household members, but also the geographical location of the household. The cost function indicates the minimum expenditure necessary for the household to attain utility level U. The cost of living index for a household in two different price regimes is simply the ratio of the two cost values, evaluated at the respective price levels. Let p^D denote domestic prices and p^A prices abroad. Calculation of ppp as a true cost of living index L is simply $L(p^D, p^A, z, U) = C(z, p^A, U)/C(z, p^D, U)$, ruling out potential differences in preferences or in the utility function for the household at home and abroad. A 'true utility based purchasing power parity' UBP instead would require the explicit inclusion of households' location (domestic D or abroad A) in the vector of household characteristics z, i.e.

$$UBP(z^A, z^D, p^D, p^A, U) = C(z^A, p^A, U)/C(z^D, p^D, U) \quad . \tag{1}$$

Alas, empirical estimation of UBP seems to be not at all promising: The approach turns out to be very data consuming as a fully parameterized cost function has to be specified and estimated. Moreover, unless experimental data were available, a serious identification problem arises due to the multicollinearity obviously inherent to observed data on z and p.[4]

However, things are simplified whenever differences between p^D and p^A can be considered as beeing minor, compared to differences between (conditional) preferences at home and abroad. With $p^D = p^A$ holding at least approximately,

in environmental endowment cause changes in the households preferences for market goods.

[4]In an experimental setting, households' would be confronted successively with p^D and p^A, with households' location remaining unchanged.

(1) simplifies to

$$S(z^A, z^D, p, U) = C(z^A, p, U)/C(z^D, p, U) \quad . \tag{2}$$

In welfare economics, this expression defines the 'equivalence scale': If income of a household characterized by z^A resembles $S(\cdot)$-times the income of household z^D ('reference household'), both of the households reach the same utility level (provided that they face identical prices). We choose the notation S like 'scale' to denote ppp at identical prices and we use the expression 'scale' synonymous with 'parity' below.

In this paper, we focus on the empirical derivation of ppp according to (2). As the exclusion of spatial variation in prices is crucial to these parities, and as prices are assumed to be constant across different regions of a particular area, we call parities defined by (2) 'regional ppp'. Hence, the resulting parities may not be applied to comparisons where 'abroad' is 'far away', but 'regional ppp' prove to be meaningful whenever their application is restricted to areas within a homogenic economic territory where differences in prices are less pronounced.[5] In Section 2 a theoretical model of regional ppp is developed. This model accomodates the fundamental identification problem as discussed in the equivalence scale literature. A convenient parametrical specification for the model is presented in Section 3 and specification tests are suggested in Section 4. Results of an empirical application to compare purchasing power in East and West Germany in 1998 are discussed in Section 5. Conclusions are drawn in the final section of the paper. Our study is related to the work of Ferrari (2003), who recently used a similar model to compare purchasing power in 19 regions of Italy. However, due to a richer data base, we are able to abstain from numerous parametrical restrictions necessary in Ferraris model. Moreover, we focus on procedures for model validation, whereas Ferrari does not report any results of specification tests.

[5] Hence, our approach may be appropriate for deriving 'Germany-Sweden ppp' or 'Germany-Spain ppp', but not for calculating a 'Germany-India ppp'.

2 Parities independent of base utility

As is well known from the analysis of equivalence scales, deriving parities according to (2) is not possible without additional identifying assumptions, as utility U is an ordinal, not a cardinal measure.[6] One solution to overcome the identification problem is to postulate that scales do not vary with the utility level. This solution was developed independently by Blackorby and Donaldson (1988) and Lewbel (1989). This assumption of 'independence of base utility (IB)' implies that the cost function can be factorized according to $C(z, p, U) = m(z, p)\tilde{C}(p, U)$. Hence, with the IB-assumption, the utility dependent cost factor is the same for both households and the parity (2) reduces to $S(z^A, z^D, p, U) = m(z^A, p)/m(z^D, p)$. Let the domestic household be the 'reference household' and use the normalization $m(z^D, p) = 1$. Then the parity reads $S(z^A, z^D, p, U) = m(z^A, p)$

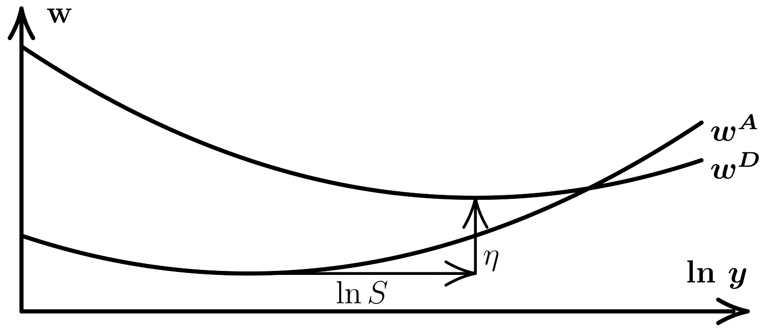
Note that the IB assumption is not testable by statistical procedures. However, IB restricts the (shape of the) resulting demand curves. These restrictions can be interpreted as necessary conditions for base independence, and they can be tested using household expenditure data. Obviously, parities estimated under the IB assumption are not reliable whenever the data do not conform to the restrictions implied by IB.

Implications of the IB assumption for households' income-expenditure relations (Engel curves) are clearly derived by Blundell et al. (1998), starting from the (log) indirect utility function. As we restrict our analysis to two households[7] characterized by z^A and z^D, respectively, and parities are assumed to be independent of utility, we shortly write $S(z^A, z^D, p, U) = S(p)$. Recalling the definition of the parity (or scale) $S(p)$, indirect utility of both of the households

[6] Any strictly increasing transformation of U leaves households' observed demand behavior unchanged. However, as the tranformation may depend on household characteristics z, the scale may be affected by the particular transformation. Hence, equivalence scales can not be identified from market data alone. See Blundell et al. (1994).

[7] See footnote 10 concerning an extension of the analysis allowing for more than two households/countries.

Figure 1: Shape invariance and log-quadratic Engel curves



is linked by

$$\ln V(\ln p, \ln y, z^A) = \ln V(\ln p, \ln y - \ln S(\ln p), z^D) \quad (3)$$

With m commodity groups, p denotes a vector of m prices, q indicates quantities and y is household income.[8] Budget share demand equations for each commodity group i, $i = 1, \ldots, m$ are derived using Roy's identity

$$w_i = -\frac{\partial \ln V(\cdot)/\partial \ln p_i}{\partial \ln V(\cdot)/\partial \ln y} \quad , \qquad w_i = \frac{p_i q_i}{y} \quad . \quad (4)$$

Applying (4) to (3), the resulting budget share Engel curves are

$$w_i(\ln p, \ln y, z^A) = w_i(\ln p, \ln y - \ln S(\ln p), z^D) + \eta_i(\ln p) \quad \text{i=1, \ldots, m.} \quad (5)$$

$\eta_i(\ln p) = \partial \ln S(\cdot)/\partial \ln p_i$ denotes the price elasticity of the parity (scale). Geometrically, (5) states that in $(w_i \times \ln y)$ space, budget share Engel curves for households A and D can be made congruent merely by horizontal and vertical shifts, a property frequently called 'shape invariance'(Pendakur (1999)). The size of the horizontal shift measures the (log) parity of both households, $\ln S$. Figure 1 gives an illustration in case of log-quadratic budget share Engel curves.

[8]Due to the static character of the model we abstract from households' savings and use the term 'income' synonymous with 'total expenditure'.

3 A quadratic model of base independent parities

Note that inference in the IB model (5) does not call for a particular functional form of households' Engel curves. Pendakur (1999) uses a semiparametric framework to derive equivalence scales from (5) for British households. However, as (5) requires a single scale, i.e. an identical horizontal shift of Engel curves for all commodities, a theoretically consistent application of semi- or nonparametric approaches turns out to be cumbersome if not impossible, as it is not clear how cross equation restrictions (across commodity groups) can be incorporated in the analysis.

Fortunately, the log-quadratic specification of budget share Engel curves proves to be a convenient parametric approach to analyze IB parities. This specification conforms to neoclassical demand theory and can be interpreted as a quadratic generalization of the Almost Ideal Demand System, see Banks et al. (1997).

To simplify notation further, we start with considering a single commodity group. This allows suppressing commodity index i. Moreover, we shortly write S instead of $S(p)$. According to (5), IB specification in a log-quadratic model leads to

$$w^D = \beta_1(\ln y - \beta_2)^2 + \beta_0 \tag{6}$$

$$w^A = \beta_1(\ln y - \beta_2 - \ln S)^2 + \beta_0 + \eta \quad , \tag{7}$$

The (observable) reduced form corresponding to (6) and (7) is

$$\begin{aligned} w^D &= c^D(\ln y)^2 + b^D \ln y + a^D \\ w^A &= c^A(\ln y)^2 + b^A \ln y + a^A \quad . \end{aligned} \tag{8}$$

Structural and reduced form coefficients are linked by the following relations:

$$c^D = \beta_1, \quad c^A = \beta_1, \quad b^A = -2\beta_1\beta_2, \quad a^D = \beta_1{\beta_2}^2 + \beta_0,$$

$$b^A = -2c^D \ln S + b^D$$

and

$$a^A = c^D (\ln S)^2 - b^D \ln S + a^D + \eta_i \quad .$$

With $c^A = c^D = c (= \beta_1)$, the parity is given by[9,10]

$$\ln S = \ln S_{D,A} = \frac{b^D - b^A}{2c} \quad . \tag{10}$$

4 Specification tests

Empirical validity of the log-quadratic specification can be tested ucing procedures that compare nonparametric Engel curve estimates with their parametric counterparts.

Testing consequences of the IB assumption in the log-quadratic framework is straightforward. So far we have suppressed the commodity index, regarding a single commodity group. However, analysis can be easily extended to m commodity groups. As base independence requires budget share Engel curves to exhibit shape invariance for each commodity group,

$$c_i^A = c_i^D = c_i \quad i = 1, \ldots, m \tag{11}$$

is a necessary condition for base independence. Moreover, under base independence parities have to be identical for each commodity group. Hence $(b_1^D - b_1^A)/(2c_1) = (b_2^D - b_2^A)/(2c_2)$ is required, or, in general,

$$b_i^A = b_i^D - \frac{c_i}{c_1}(b_1^D - b_1^A) \quad i = 2, \ldots, m \quad . \tag{12}$$

[9] The price elasticity of the parity can be calculated as

$$\eta = \eta_{A,D} = a^A - a^D + b^D \ln S_{D,A} - c (\ln S_{D,A})^2 \quad . \tag{9}$$

[10] Equation (10) reveals that in the log-quadratic specification, parities for more than two countries prove to be transitive by construction. For a third country (index T) $S_{D,T} = S_{D,A} \cdot S_{A,T}$, as base independence requires $c^A = c^D = c^T = c$, and $\ln S_{D,A} = \frac{(b^D - b^T) - (b^A - b^T)}{2c} = \ln S_{D,T} - \ln S_{A,T}$ according to (10).

(11) restricts the curvature and (12) additionally restricts the horizontal shifts of Engel curves to be identical across commodity groups. (11) poses m restrictions, and (12) $m-1$ further parameter restrictions on the system of Engel curves. Hence, we are able to test the base independence assumption or shape invariance hypothesis

$$H^{IB}: \quad \text{Both (11) and (12) do hold.} \tag{13}$$

using conventional parameter tests. Note that the 'curvature condition' and the 'shift condition' may be tested separately. Obviously, the 'curvature condition' reads

$$H^{curv.}: \text{(11) holds.} \tag{14}$$

For the 'shift condition', (12) has to be recalculated allowing for $c_i^A \neq c_i^D$. This results in

$$b_i^A = \frac{c_i^A}{c_i^D} b_i^D - c_i^A \frac{c_1^A b_1^D - c_1^D b_1^A}{c_1^A c_1^D} \quad i = 2, \ldots, m \quad . \tag{15}$$

Hence, the 'shift condition' implies the hypothesis

$$H^{shift}: \text{(15) holds.} \tag{16}$$

5 Empirical application: Regional ppp for East and West Germany in 1998

In our empirical application we aim at calculating ppp for East and West Germany, i.e. the FGR excluding the territory of the former GDR (WEST) and the New Laender (EAST, territory of the former GDR). This comparison is of particular interest, as for instance after the reunification separate wage regimes were observed in both the public and the private sector as well, with lower nominal wages in the East. As prices also were generally lower in the New Laender, the sign of differences in real wages was indeterminate. With

Table 1: Household types and sample sizes

Household Type		Number of observations WEST	Number of observations EAST
Single, 18-64 years old	S0	6492	1383
Married Couple, both 18-64 years old, no children	C0	6268	1891
Married Couple, 2 children, both children younger than 18 years old	C2	5540	1304

wage differentials continuing to exist (mainly due to a still lower productivity in East Germany), while prices in both regions roughly resemble each other in the end of the 1990s, it remains an open question as to whether the standard of living in both of the regions differed, or whether income inequality was offset by differences in the regional utility based purchasing power. Another question of particular interest is whether an analysis along the lines described above leads to a regional ppp of 1. Merely in this case assuming a 'representative German consumer' in empirical aggregate (macroeconomic) models for Germany is justified.

We draw on a 80%-sample of the 1998 German Income and Expenditure Survey. As preference shifts may differ for households of different demographic composition, we apply the model separately to data for three household types: Single households, childless married couples and married couples with two children. Sample sizes are given in Table 1. Household expenditure is analyzed for 6 commodity groups: FOOD, CLOTHING, SHELTER, MOBILITY (including communication), EDUCATION (including leisure) and OTHERS.[11,12]

In a first step we estimated both unrestricted log-quadratic budget share En-

[11]When estimating the system of Engel curves, the equation for OTHERS was dropped to account for the system's adding-up constraint.

[12]Note that for SHELTER, the assumption of identical prices in the EAST and in the WEST is crucial. However, it is extremely difficult to disentangle mere price differences and quality differences concerning housing in both of the regions.

Table 2: Test results for log-quadratic Engel curves

	WEST			EAST		
Commodity	S0	C0	C2	S0	C0	C2
FOOD					*	
CLOTHING					*	*
SHELTER			*			
MOBILITY	*	*	*	*		
EDUCATION		*		*		
OTHERS	*	*				

The asterisk * indicates rejection of the log-quadratic functional form hypothesis as compared to a nonparametric alternative with $\alpha = 0.05$

gel curves and nonparametric Engel curves as well, broken down by regions, householdtypes and commodity groups. For nonparametric estimation, the Nadaraya-Watson estimator was employed using a quartic kernel and the optimal bandwidth according to crossvalidation. Nonparametric estimation results are displayed in the appendix.

The assumption concerning functional (log-quadratic) form of Engel curves was tested using the 'smooth-conditional-moments, SCM' technique proposed by Gozalo (1997). The test procedure is based on the difference between parametric and nonparametric estimates, and the SCM bootstrap procedure was based on 1000 replications. Results of the SCM bootstrap test are given in Table 2. With the exception of MOBILITY in West Germany there is no systematic rejection of the log-quadratic functional form for Engel curves in the sample. Even though there are rejections for particular commodities for each of the household types, we consider the log-quadratic as an adequate 'work horse' for the analysis to follow.

Estimated parities for the three household types under study are presented in Table 3. WEST was used as the reference region (domestic, D). Note that estimation was carried out for the three household types separately. Hence, it

Table 3: Estimated base independent purchasing power parities for East Germany

Household Type	Estimate	Standard error
S0	0.851	0.0345
C0	0.754	0.0280
C2	0.779	0.0282

is remarkable that the values resemble each other, at roughly 0.8. The fact that each estimated value turns out to be significantly different from 1, indicates pronounced differences in consumption patterns in the East and the West of Germany in 1998.[13]

An estimated parity of 0.8 for East Germany means, that a household moving from the west to the east would require only 80% of the income he has in the west, to be equally well off at both of the locations. This estimate is rather low and could be used to justify wage differences in the East and the West from a cost of living perspective only if the model proves to be reliable, i. e. compatible with the data used.

Unfortunatly, this turns out to be not the case. As indicated in Table 4 a), the IB assumption has to be clearly rejected for all of the three household types. Testing for curvature and identical shifts separately shows that it is mainly the assumption of an identic curvature of budget share Engel curves in the East and the West that leads to rejection of IB and therefore of the model, see Table 4 c)). The structure of consumption differences in both parts of Germany are enlighted by the results given in Table 4 b), indicating that the shift condition is not strongly rejected for S0 and C2.

[13] We also evaluated data of the 1993 German Income and Expenditure Survey. In 1993, results were more favourable for the log-quadratic functional form. Alas, the model failed specification tests. Therefore, we do not report detailed results here. It should be mentioned, however, that estimates for utility based parity came to 0.410, 0.435 and 0.557 for S0, C0 and C2, respectively. In conjunction with the results displayed in Table 3, these estimates indicate the process of converging consumption patterns in the East and the West of Germany.

Table 4: Results of model specification tests (Likelihood-Ratio-Tests)

a) Test of H^{IB}: Base independence (shape invariance), see (13)

Household type	Test statistic	p-value
S0	50.86	0.000
C0	98.00	0.000
C2	46.37	0.000

b) Test of H^{shift}: Identical horizontal shifts of Engel curves, see (16)

Household type	Test statistic	p-value
S0	8.58	0.127
C0	22.50	0.000
C2	12.00	0.035

c) Test of H^{shift}: Identical curvature of Engel curves, see (14)

Household type	Test statistic	p-value
S0	27.09	0.000
C0	53.39	0.000
C2	21.45	0.002

6 Conclusion

Whenever ppp are used to compare 'true cost of living' at home and abroad, particular measures of welfare or utility have to be applied. Ordinality of the utility concept requires additional identifying assumptions, whenever empirical analysis is based only on market data. Hence, estimated parities are reliable only if validity of these assumptions is tested for (at least for those assumptions that are testable at all).

The model presented in this paper is theoretically consistent as it is derived immediately from neoclassical demand theory. We have focussed on procedures for testing model specification and we have shown how these tests work in an empirical application. As the model had to be rejected for a ppp comparison concerning the East and the West of Germany in 1998, it is of particular interest to apply the model to the data of the 2003 German income and expenditure survey. As differences in the consumption patterns of households living in both of these regions are discarded in the course of the analysis, this investigation could also reveal whether the existence of a 'representative German consumer' can be accepted in the meantime.

As our model is 'regional' in that it calls for identical prices in the territories analyzed, an obvious field of application can be seen in modelling parities for countries in the European Community, as parities of such kind could be used when deciding on particular schemes of transfer payments within the EC. In addition, harmonization of national European household surveys will provide a rich and suitable data base for these applications of the model.

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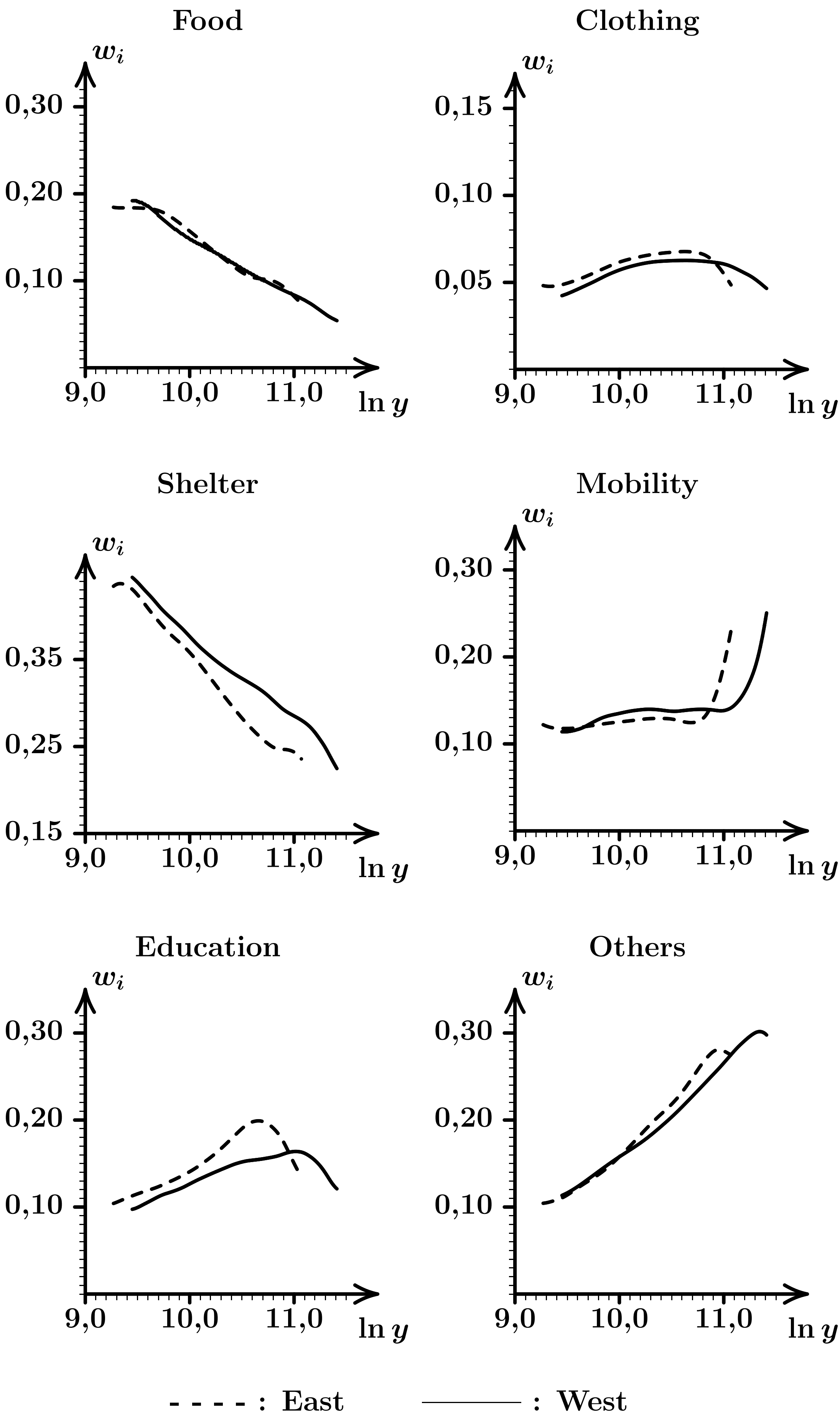
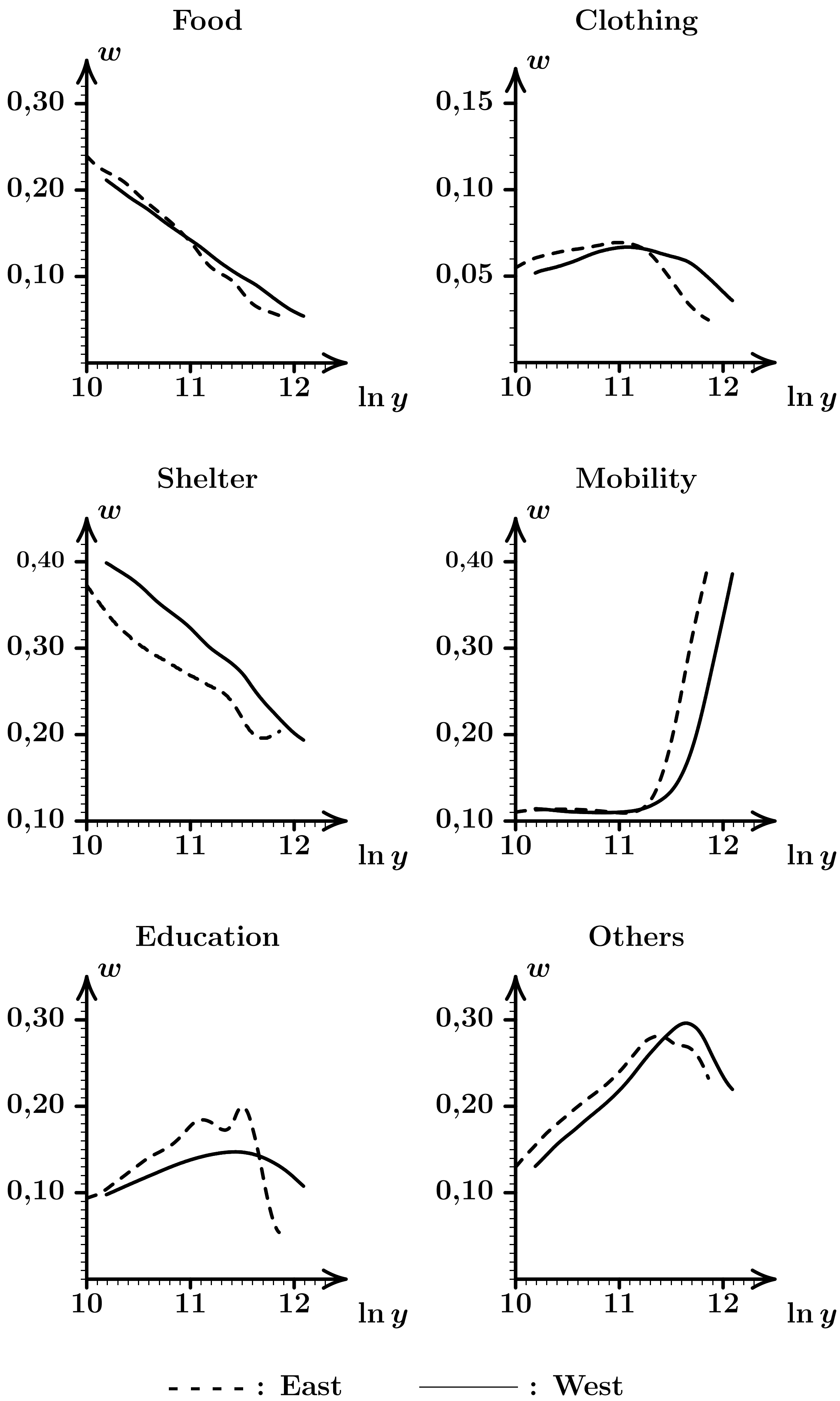
Appendix

Figure A.1.: Nonparametric Engel Curves for singles (S0)



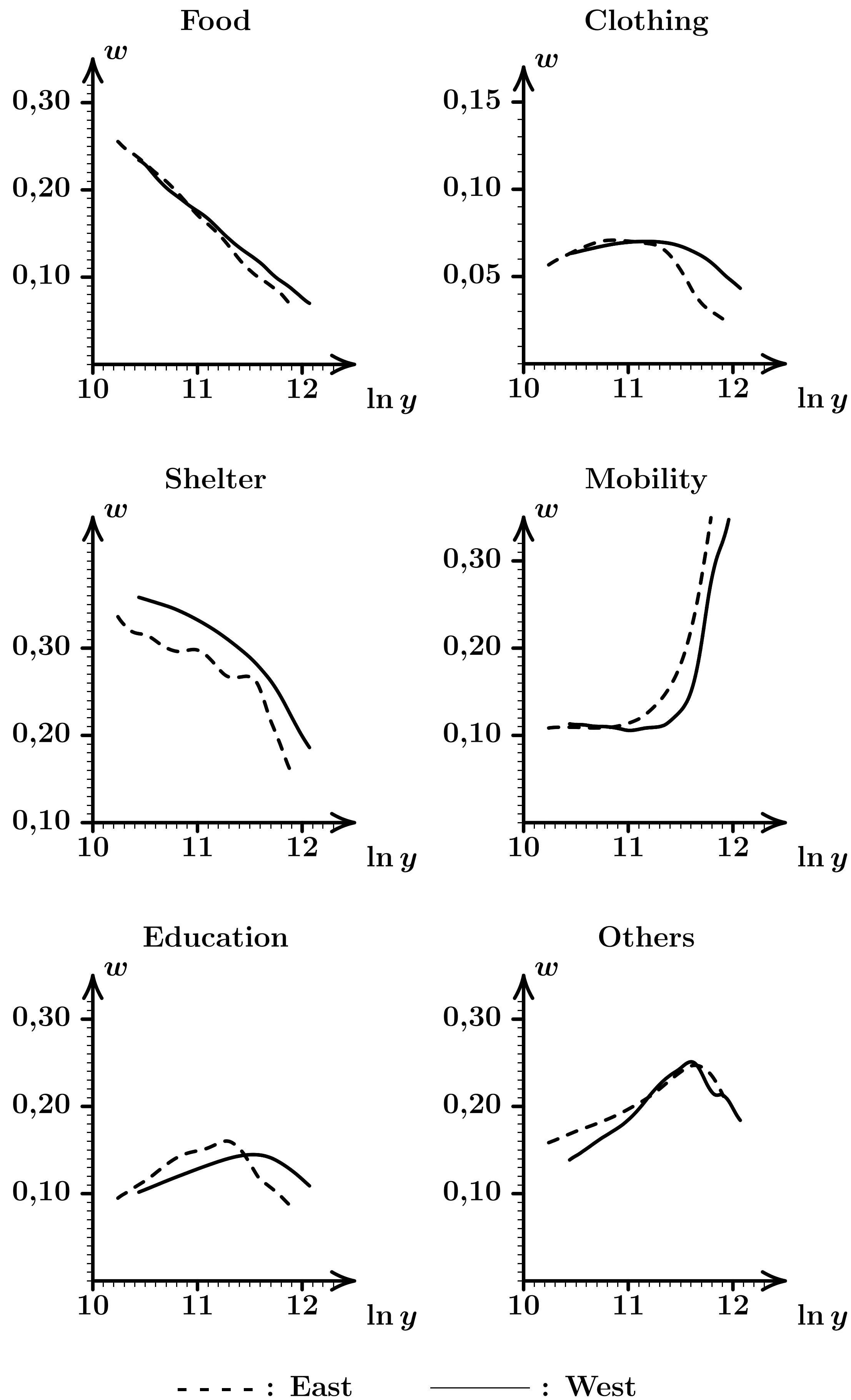
Data source: EVS 1998

Figure A.2: Nonparametric Engel Curves for married couples (no child) (C0)



Data source: EVS 1998

Figure A.3.: Nonparametric Engel Curves for married couples with 2 children (C2)



Data source: EVS 1998